Newton's Second Law for Rotation

(K. M. Westerberg, 2/2004)

Introduction

In a previous lab ("Newton's Second Law"), we verified Newton's Second Law as it applies to translational motion. It turns out that there is a very strong analogy between translational motion and rotational motion. In fact, if we make the following substitutions into the equations governing translation motion,

$$\begin{aligned}
\text{(position) } x &\rightarrow \theta \text{ (angular position)} \\
\text{(velocity) } v &\rightarrow \omega \text{ (angular velocity)} \\
\text{(acceleration) } a &\rightarrow \alpha \text{ (angular acceleration)} \\
\text{(force) } F &\rightarrow \tau \text{ (torque)} \\
\text{(mass) } m &\rightarrow I \text{ (rotational inertia)} \\
\text{(momentum) } p &\rightarrow L \text{ (angular momentum)}
\end{aligned}$$

we end up with an analogous set of equations governing rotational motion.

One of the fundamental equations which pertains to translational motion is Newton's Second Law, which relates the forces acting on a body to its acceleration ($\vec{F}_{\rm net} = m\vec{a}$). The analogous law for rotational motion is Newton's Second Law for Rotation, which relates the *torques* acting on a rigid rotating body to its *angular acceleration*. That law can be stated as follows:

$$\tau_{\rm net} = I\alpha \ , \tag{1}$$

where $\tau_{\rm net}$ is the sum of the torques acting on the body, I is its rotational inertia, and α is its angular acceleration.

In the previous lab, we verified Newton's Second Law for translational motion by connecting a hanging weight to a glider cart with a string and allowing the hanging weight to exert a force on the glider cart. We measured the glider cart's acceleration for various values of hanging weight, and plotted the tension force as a function of the glider cart's acceleration. If Newton's Second Law is valid, that graph should result in a straight line whose slope is equal to the mass of the glider cart. Friction is almost completely eliminated by allowing the glider cart to slide across a frictionless air track, although we can account for any friction with our method — any non-zero friction would show up as a non-zero y-intercept in the tension force vs. acceleration graph.

In the present lab, we will use the same method to verify Newton's Second Law for rotational motion. We will connect a hanging weight to a rotating disk wheel, thus allowing the hanging weight to exert a torque on the disk wheel. We will measure the disk wheel's angular acceleration for various values of hanging weight and plot torque as a function of the angular acceleration. If Newton's Second Law for Rotation is valid, that graph should

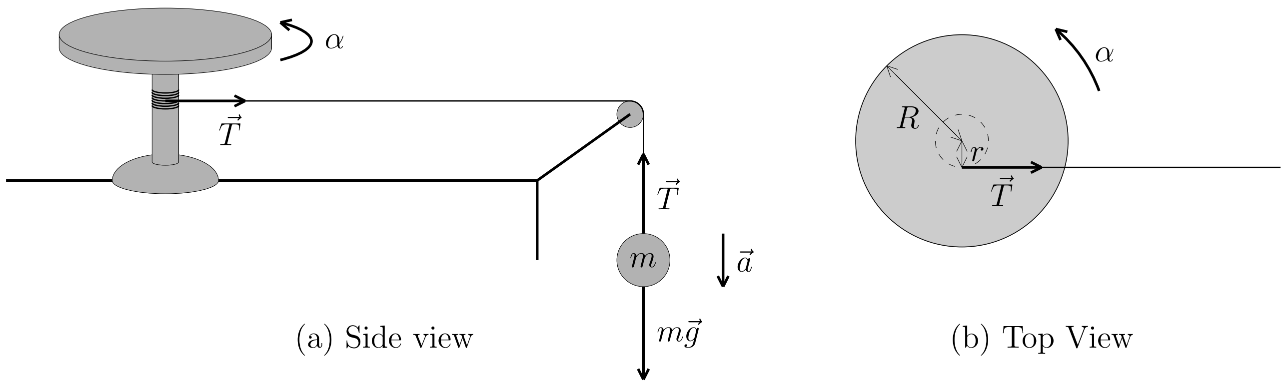


Figure 1: Disk wheel setup.

result in a straight line whose slope is equal to the rotational inertia of the disk wheel (along with the spider apparatus upon which the disk wheel rotates). Although friction is reduced by the bearings in the spider apparatus which holds the disk wheel, it cannot be eliminated. The fact that friction will play a more significant role in this lab does not present a problem, however. We will be able to account for friction by not requiring the straight line fit to pass through the origin — any non-zero friction will show up as a non-zero y-intercept in the torque vs. angular acceleration graph. The slope of the graph will not be affected by the presence of friction.

Perhaps the most significant benefit of the method we employed in verifying Newton's Second Law for translational motion is the fact that the same method can be used to verify Newton's Second Law for rotational motion. The setup and procedure of this lab are entirely analogous to the previous lab, including the method used to measure the angular acceleration of the disk wheel. As we perform this experiment, we should get the sense that we have done all of this before. Not only will we be verifying Newton's Second Law for Rotation, we should also gain much greater insight into the analogy between translational and rotational motion in general.

Setup

The basic setup is shown in Fig. 1. The disk wheel is connected to a hanging mass m using a string which is wrapped around the "drum" (i.e., the neck of the disk wheel apparatus) and passes over a pulley (the actual setup will consist of two pulleys, but that fact need not concern us now). The tension, T, in the string will exert a force on the disk wheel. This force is applied tangentially at a point which is a distance r away from the axis of rotation, where r is the drum radius, and will thus result in a torque

$$\tau_T = Tr \tag{2}$$

being applied to the disk wheel. This will cause the disk wheel to undergo an angular acceleration, which can be measured with the help of a photogate timer (not shown in Fig. 1).

As the disk wheel undergoes an angular acceleration, the hanging weight must also undergo a *linear* acceleration downward towards the floor. Although the angular acceleration of the disk wheel is not *equal* to the (linear) acceleration of the hanging weight — they don't even have the same units — the two accelerations can be related. In fact, this relationship is given by

$$a = \alpha r \ , \tag{3}$$

where a is the magnitude of the hanging weight's linear acceleration, α is the disk wheel's angular acceleration, and r is the drum radius. This relationship follows from the fact that the downward acceleration of the hanging weight must be equal in magnitude to the tangential acceleration of the string wrapped around the drum,[1] and the latter acceleration is given by αr.

Because of the downward acceleration of the hanging weight, the tension in the string is not equal to the hanging weight, but is still related to it. Applying Newton's Second Law *for translational motion* to the hanging weight (as was done in the previous lab) yields

$$T = mg - ma = mg - m\alpha r \ . \tag{4}$$

Since the angular acceleration will be measured during this experiment, all of the variables on the right-hand side of Eq. 4 will be known, and so the tension can be calculated.

Now we apply Newton's Second Law *for rotational motion* to the disk wheel. The only forces acting on the disk wheel which exert a non-zero torque are the tension force from the string and the friction force inside the rotational axis of the disk wheel. If we define the positive direction for rotation to be the direction of the angular acceleration (either clockwise or counterclockwise, depending on how the string is connected to the drum), Newton's Second Law for Rotation asserts that

$$+\tau_T - \tau_f = I\alpha \ ,$$

from which we derive

$$\tau_T = I\alpha + \tau_f \ . \tag{5}$$

If Newton's Second Law for Rotation is valid, then we must get a straight line when we plot τ_T vs. α, where the slope of the line is equal to I (the rotational inertia of the disk wheel + spider) and the y-intercept is equal to τ_f (the "frictional torque"). You should obtain a straight line which passes above the origin (τ_f should be positive and will *not* be negligible). We are, of course, assuming that the value of τ_f is independent of the hanging weight. As in the previous lab, we have no reason to believe that this is not the case.

[1] This is for the same reason that the glider cart's acceleration in the previous lab was equal in magnitude to the hanging weight's acceleration.

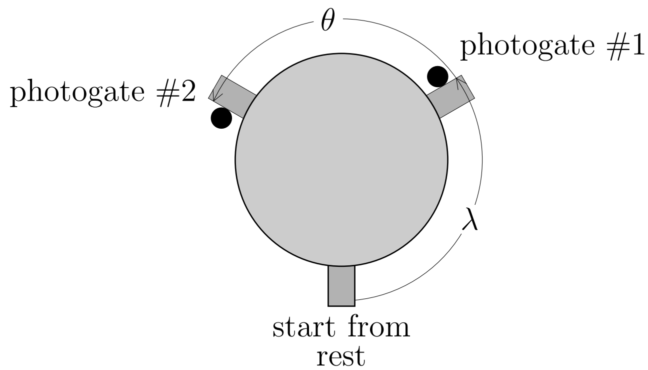


Figure 2: Schematic diagram of how one might measure the angular acceleration of the disk wheel.

Measuring the angular acceleration

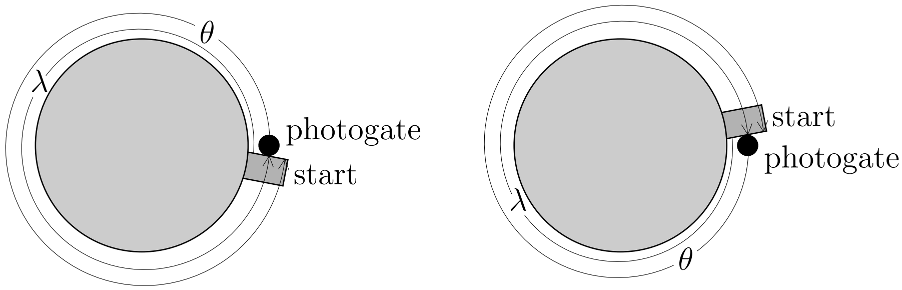
The same method used to measure the (linear) acceleration of the glider cart in the previous lab can be used to measure the angular acceleration of the disk wheel. The general setup based on this method is shown in Fig. 2. A tab is attached to the disk wheel which can be used to trigger a photogate timer. The two photogates are placed a certain *angular* distance (θ) apart, and the disk wheel is backed up behind the first photogate by a certain lead *angular* distance (λ) and released from rest. After the disk wheel rotates through an angular displacement of λ, the tab will pass through the first photogate, starting the timer. When the disk wheel rotates through an additional angular displacement θ, the tab will pass through the second photogate, stopping the timer. The timer reading can be used to calculate the angular acceleration of the disk wheel using the following equation:

$$\alpha = \frac{2(\sqrt{\lambda + \theta} - \sqrt{\lambda})^2}{(\Delta t)^2} \; . \tag{6}$$

Eq. 6 is entirely analogous to Eq. 5 in the previous lab, and is derived in exactly the same way. We will also be using the same shortcut as before — calculating the numerator once at the beginning of the lab (after the values of λ and θ have been determined) and then dividing the previously computed numerator by $(\Delta t)^2$ each time we wish to calculate the angular acceleration of the disk wheel.

Of course, we could have simplified the method by releasing the disk wheel from rest just before it triggers the first photogate, effectively setting λ equal to zero. However, this will lead to all sorts of difficulties that have already been discussed in the previous lab: the disk wheel would be rotating very slowly as it approaches the first photogate, and would take an unexpectedly large amount of time to reach it. A modest error in where we release the disk wheel would result in very large errors in calculating its angular acceleration. Giving the disk wheel a significant running start should eliminate this problem.

The equipment required to measure the angular acceleration of the disk wheel as shown in



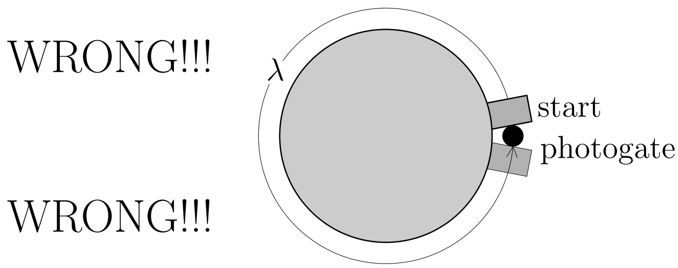
(a) counterclockwise rotation (a) clockwise rotation

Figure 3: How we will measure the angular acceleration in this lab. Each lab group should take note of which direction their disk wheel has been set up to rotate, and use the appropriate figure above as a guide. The disk wheel should be released from rest so that the tab is just about to trigger the photogate. Timing will begin the *second* time through the photogate, so that λ and θ are both equal to 1 rev.

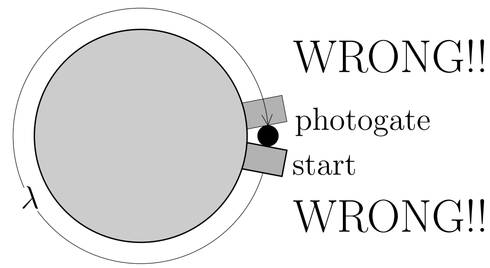
Fig. 2 includes two photogates and a protractor (or similar device) for measuring λ and θ. Not only are photogates expensive, but measuring the angular displacements using a protractor could prove to be very clumsy. It turns out that we really only need one photogate and *no protractors*, thanks to one property of rotational motion which is *not* shared by translational motion — *rotational motion is periodic.* Whenever the disk wheel rotates through a 360° angle, it returns to its original position. We can take advantage of this fact.

Fig. 3 shows the setup we will actually use for this lab. For the convenience of the student, both clockwise and counterclockwise rotation are shown — you should follow the picture which corresponds to the direction that your disk wheel will start rotating when it is released (that direction depends on which way the string is wrapped around the drum). In either case, the disk wheel should be aligned with the photogate and released from rest just *before* it is about to trigger the photogate. You can determine this release position for each run by actually rotating the disk wheel slowly towards the photogate until the photogate is triggered (this is entirely analogous to the method used in the previous lab for determining the position of the glider cart when it triggered each of the two photogates). You will then back the disk wheel up a very small amount and release it *from rest.* The disk wheel should pass through the photogate almost immediately after you release it.

At this point, it may seem to you that we are recommending a lead angular displacement of zero — we are releasing the disk wheel right next to the photogate. Actually, this is not true. After the tab has passed through the photogate for the first time, *stop and clear the photogate timer.* You will have plenty of time to do this, since the disk wheel will be rotating very slowly through its first revolution. The timer will be started for real when the tab passes through the photogate *for the second time*, exactly one revolution after the disk wheel has been released. The disk wheel will then proceed to rotate with the timer running until the tab passes through the photogate *for the third time*, which will stop the timer. At this point,



(a) counterclockwise rotation



(a) clockwise rotation

Figure 4: Above is an example of what *not* to do. This figure should be contrasted with Fig. 3. Note that in this figure, the disk wheel is released from rest with the tab on the wrong side of the photogate. The lead angular displacement (λ) will be less than 1 rev, which will lead to errors in the measurement of angular acceleration.

you should catch the disk wheel to prevent the tab from passing through the photogate for a fourth time, starting the timer again. The time you read on the photogate timer can be used to calculate the angular acceleration of the disk wheel. Values of λ and θ do not need to be measured — both angular displacements will be exactly equal to 1 rev. All you have to do is convert this to radians (the SI unit) and calculate the acceleration numerator at the start of the lab.

Fig. 4 illustrates a mistake students often make when performing this lab. It concerns the initial position of the disk wheel when it is released from rest. Suppose for the sake of argument that the hanging weight is connected to the disk wheel in such a way that the disk wheel will rotate counterclockwise when it is released. This means that the disk wheel must be rotated *clockwise* in order to raise the hanging weight again for the next run. It is tempting to rotate the disk wheel until the tab is about to go through the photogate *in the clockwise direction*, and then release it from rest from that position. However, this will result in the lead angular displacement being less than 1 rev, since the disk wheel will be positioned in such a way that it has already passed through the photogate *in the counterclockwise direction* (see Fig. 4(a)). Therefore, *when rewinding the disk wheel in preparation for the next run, make sure you rotate the disk wheel so that the tab passes through the photogate all the way, and then rotate the disk wheel in the opposite direction, approaching the photogate in the forward direction.* This will ensure that the disk wheel approaches the photogate from the correct side, and that the lead angular displacement will be correct.

Of course, if your disk wheel is connected to the hanging weight in such a way that it rotates clockwise after being released, simply reverse all of the directions above. The instructions are basically the same: make sure the disk wheel approaches the photogate from the correct side before releasing it from rest. *If you do not understand the preceding two paragraphs or the difference between Figs. 3 and 4, ask your instructor for help.*

Procedure

The basic procedure of this lab is to connect the disk wheel to a hanging weight, measure its angular acceleration for various values of hanging weight, and plot torque versus angular acceleration. The resulting graph should be a straight line with a slope equal to the rotational inertia of the disk wheel + spider and a y-intercept equal to the frictional torque acting on the disk wheel. The rotational inertia obtained from the graph will be compared to the theoretical rotational inertia which can be calculated from the disk wheel's mass and diameter using one of the standard formulas for rotational inertia found in your textbook.

The following procedure should be followed.

(Q-1) Measure the diameter of the drum with Vernier calipers and calculate the drum radius from this value.

The drum radius is the value of r which you will need to calculate the torque and the downward (linear) acceleration of the hanging weight.

(Q-2) Determine the lead angular distance (λ) and the angular distance over which the time is measured (θ), and calculate the numerator for the angular acceleration.

If you plan to follow the directions given in the previous section, λ and θ will both be equal to 1 rev. If your instructor gives alternative directions, follow those instead. Be sure to express λ and θ in radians (the SI unit).

(Q-3) Perform the experiment and fill in the data table provided.

You should start off with a hanging mass of 50 g (0.050 kg). Rotate the disk wheel to its release position right next to the photogate, as explained in detail last section (make sure the disk wheel approaches the photogate from the correct side), and then release it from rest. As soon as the disk wheel passes through the photogate the first time, stop and clear the timer. Without interfering in any way with the rotation of the disk wheel, allow it to undergo two full revolutions after its release. The disk wheel should start the timer for real after the first revolution (when the tab triggers the photogate for the *second* time), and stop the timer after the second revolution. After the timer is stopped, catch the disk wheel to prevent it from starting the timer again. Record the photogate time in the data table provided. You will take five measurements for each value of hanging mass and calculate the average time — this is your value of Δt. Use this value of Δt to calculate the angular acceleration of the disk wheel. Repeat this procedure for the other hanging masses listed in the table.

This should all sound very familiar.

(Q-4) Continue the analysis by filling in the second data table.

You should know how to calculate the hanging weight from the hanging mass. The angular acceleration can be copied from the previous data table. The tension can be calculated using Eq. 4, and the torque can be calculated using Eq. 2.

(Q-5) Plot torque vs. angular acceleration and determine the experimental value of the rotational inertia of the disk wheel + spider (I) and the frictional torque (τ_f).

Show how you do this. Be sure to calculate the uncertainty for I.

(Q-6) Calculate the theoretical value for the rotational inertia of the disk wheel + spider.

The mass of the disk wheel is engraved in the wheel itself. A reasonable estimate for its uncertainty can be obtained by considering the number of significant figures reported. The diameter can be measured using a meter stick with the caliper jaws. Its uncertainty can be obtained from Table 1 in "Measurement and Calculation". The rotational inertia of the disk wheel alone should now be calculated using a formula you should be able to find in your textbook. Show clearly how you calculate the uncertainty of this quantity. The spider can be accounted for by adding $0.002\,\mathrm{kg\,m^2}$ to the rotational inertia.

(Q-8) Compare the experimental value of I obtained from the graph with the theoretical value obtained from the calculation performed in (Q-7) using the discrepancy test.

Don't forget to clean up the lab station when you are finished.